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Erratum

Obliquely incident surface waves in shallow water of slowly varying depth

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The analysis below Fig. 1 in Sect. 3 of the original paper is not correct. Here, a revised version is presented. This should replace the original analysis at the end of Sect. 3.

The composite shelf, induced by the primary wave, cannot propagate further than the characteristics $\theta_+ = 0,\ \theta_- = 0$. In other words, this shelf is confined to a wedge-shaped region in the (ζ, ξ)-plane, bounded above by the characteristics (Fig. 1). Accordingly, Eq. (44) is integrated along C between the initial point A and the end point B. This leads to the equation

$$\int_{\hat{\zeta}(\tau)}^{\bar{\zeta}(\tau)} \hat{\eta}_\tau(\zeta, \xi_0, \tau)\mathrm{d}\zeta + \int_{\hat{\zeta}(\tau)}^{\bar{\zeta}(\tau)} h\hat{\phi}_{\xi\xi}(\zeta, \xi_0, \tau)\mathrm{d}\zeta + [h\hat{\phi}_\zeta]_+ - [\hat{\phi}_\zeta]_-$$

$$= \frac{\mathrm{d}}{\mathrm{d}\tau}\int_{\hat{\zeta}(\tau)}^{\bar{\zeta}(\tau)} \hat{\eta}(\zeta, \xi_0, \tau)\mathrm{d}\zeta + \int_{\hat{\zeta}(\tau)}^{\bar{\zeta}(\tau)} h\hat{\phi}_{\xi\xi}(\zeta, \xi_0, \tau)\mathrm{d}\zeta + [h\hat{\phi}_\zeta - k^{-1}\hat{\eta}]_+ - [\hat{\phi}_\zeta + k_0^{-1}\hat{\eta}]_- = 0, \tag{47}$$

where $[..]_+$ and $[..]_-$ denote evaluation along the characteristics $\theta_+ = 0$ and $\theta_- = 0$, respectively.

Since the contribution of the right-going shelf (41) to mass transfer is negligibly small, the mass loss of the primary wave is completely transferred to the composite shelf. Based on a comparison of (32), (40) and (47) it is then found that the requirement that mass is conserved necessarily implies that

$$[h\hat{\phi}_\zeta - k^{-1}\hat{\eta}]_+ - [\hat{\phi}_\zeta + k_0^{-1}\hat{\eta}]_- = m_0 k_0^{3/2} h_+ k_+^2 \frac{\mathrm{d}}{\mathrm{d}\tau}(h_+^{-1/2} k_+^{-3/2}). \tag{48}$$

When h and k are expressed in terms of the variables θ_+ and θ_-, it is found that these are of the form $h = \hat{h}(\theta_- - \theta_+)$, $k = \hat{k}(\theta_- - \theta_+)$, which implies that $h_+ = \hat{h}(\theta_-)$, $k_+ = \hat{k}(\theta_-)$. Clearly, the right-hand side of Eq. (48), with $\mathrm{d}/\mathrm{d}\tau = -2\mathrm{d}/\mathrm{d}\theta_-$, is independent of θ_+. The first term on the left-hand side of Eq. (48), with $\hat{\phi}$ and $\hat{\eta}$ expressed in terms of θ_+ and θ_-, is also independent of θ_+. This leads to the conclusion that the second term is of the form

$$[\hat{\phi}_\zeta + k_0^{-1}\hat{\eta}]_- = k_0^{-1}[(1 + k_0^2)\hat{\phi}_{\theta_+} + \ell^2\hat{\phi}_{\theta_-}]_- = m_0 F_0, \tag{49}$$

where the constant F_0 is still to be determined.

From (48) and (49) we obtain

$$[h\ell^2\hat{\phi}_{\theta_+} + (1+hk^2)\hat{\phi}_{\theta_-}]_+ = -m_0 k_0^{3/2} h_+ k_+^3 \frac{\mathrm{d}}{\mathrm{d}\tau}(h_+^{-1/2} k_+^{-3/2}) - m_0 k_+ F_0. \tag{50}$$

It is assumed that $\mathrm{d}h/\mathrm{d}\zeta$ is continuous. Then $\mathrm{d}h/\mathrm{d}\zeta = 0$ at $\zeta = 0$, and the first term on the right-hand side of Eq. (50) vanishes as $\tau \downarrow 0$ if the path of integration is chosen along the ζ-axis. Since this term is evaluated at $\theta_+ = 0$, and $\overline{\zeta}(\tau) \downarrow 0$ as $\tau \downarrow 0$, this corresponds to the limit $\theta_- \uparrow 0$. From (49) and (50) it then follows that

$$\hat{\phi}_{\theta_-} = -m_0 F_0/2k_0, \quad \hat{\phi}_{\theta_+} = m_0 F_0/2k_0 \quad \text{at} \quad \theta_+ = 0, \quad \theta_- = 0, \tag{51}$$

which implies that $\hat{\eta} = 0$ at $\theta_+ = 0$, $\theta_- = 0$. In addition, the right-going wave component of the composite shelf should vanish in the region $\zeta < 0$, i.e., $\hat{\eta}_{\theta_+} = 0$ in the region of uniform depth. Combining these results, we obtain

$$[\hat{\eta}]_- = 0. \tag{52}$$

The equation for the potential, derived from (43)–(45), is of the form

$$\hat{\phi}_{\theta_+\theta_-} = (4hk)^{-1}[(hk)_{\theta_+} - (hk)_{\theta_-}](\hat{\phi}_{\theta_+} - \hat{\phi}_{\theta_-}). \tag{53}$$

Along the characteristic $\theta_+ = 0$, Eq. (53) converts into an ordinary differential equation. Using (50) to eliminate the term $[\hat{\phi}_{\theta_-}]_+$, the resulting equation reads

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}\theta_-}[\hat{\phi}_{\theta_+}]_+ &+ (2h_+k_+(1+h_+k_+^2))^{-1}\frac{\mathrm{d}}{\mathrm{d}\theta_-}(h_+k_+)[2[\hat{\phi}_{\theta_+}]_+ \\ &+ m_0 k_+ F_0 - 2m_0 k_0^{3/2} h_+ k_+^3 \frac{\mathrm{d}}{\mathrm{d}\theta_-}(h_+^{-1/2}k_+^{-3/2})] = 0.\end{aligned} \tag{54}$$

Writing Eq. (54) in the form $\mathrm{d}[\hat{\phi}_{\theta_+}]_+/\mathrm{d}\theta_- + \alpha(\theta_-)[\hat{\phi}_{\theta_+}]_+ + \beta(\theta_-) = 0$, the initial condition (51) leads to the solution

$$[\hat{\phi}_{\theta_+}]_+ = \left[m_0F_0/2k_0 - \int_0^{\theta_-} \beta(s)\exp\left(\int_0^s \alpha(u)\mathrm{d}u\right)\mathrm{d}s\right]\exp\left(-\int_0^{\theta_-}\alpha(s)\mathrm{d}s\right). \tag{55}$$

The expression for $[\hat{\phi}_{\theta_-}]_+$ is determined from (50) and (55). This leads to the result

$$\begin{aligned}[\hat{\eta}]_+ = (1+h_+k_+^2)^{-1}&[2m_0k_0^{3/2}h_+k_+^3\frac{\mathrm{d}}{\mathrm{d}\theta_-}(h_+^{-1/2}k_+^{-3/2}) - m_0k_+F_0] \\ &+ \frac{2h_+k_+^2}{(1+h_+k_+^2)}\left[m_0F_0/2k_0 - \int_0^{\theta_-}\beta(s)\exp\left(\int_0^s\alpha(u)\mathrm{d}u\right)\mathrm{d}s\right]\exp\left(-\int_0^{\theta_-}\alpha(s)\mathrm{d}s\right).\end{aligned} \tag{56}$$

The mass carried by the composite shelf along C, expressed by the integral

$$\int_{\hat{\zeta}(\tau)}^{\overline{\zeta}(\tau)} \hat{\eta}(\zeta, \xi_0, \tau)\mathrm{d}\zeta, \tag{57}$$

must be bounded as $\tau \to \infty$. This necessarily implies that $[\hat{\eta}]_+ \to 0$ in this limit, which corresponds to the requirement that $[\hat{\eta}]_+ \to 0$ as $\theta_- \to -\infty$. Then the constant F_0 in (56) is uniquely determined.

The requirement that mass is conserved leads to the expressions (52) and (56) for $[\hat{\eta}]_-$ and $[\hat{\eta}]_+$, which prescribe the solution of Eq. (46) along the characteristics $\theta_- = 0$ and $\theta_+ = 0$, respectively. The problem thus posed has a unique solution [16]. Finally, it is noted that the physically relevant region is confined to the third quadrant in the (θ_-, θ_+)-plane.

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